A Sparse-Matrix/Canonical Grid Method for Analyzing Densely Packed Interconnects

Shu-Qing Li, *Member, IEEE*, Yongxue Yu, Chi Hou Chan, *Senior Member, IEEE*, Ka Fai Chan, and Leung Tsang, *Fellow, IEEE*

***Abstract*—In this paper, a fast numerical method called the sparse-matrix/canonical-grid (SM/CG) method is employed to analyze densely packed microstrip interconnects that involve a large number of unknowns. The mixed-potential integral equation is solved by using the method of moments in the spatial domain. The closed-form expressions of the spatial Green's functions of microstrip structures are obtained from the combination of the fast Hankel transform and the matrix pencil method. The Rao–Wilton–Glisson triangular basis functions are used to convert the integral equation into a matrix equation. The matrix equation is then solved by using the SM/CG method, in which the far-interaction portion of the matrix–vector multiplication in the iterative solution is performed by the fast Fourier transforms (FFTs). This is achieved by the Taylor series expansions of the spatial Green's functions about the uniformly spaced canonical grid points overlaying the triangular discretization. Numerical examples are presented to illustrate the accuracy and efficiency of the proposed method. The SM/CG method has computational complexity of $O(N \log N)$. Furthermore, being FFT-based facilitates the implementation for parallel computation.**

***Index Terms*—Fast Hankel transform, interconnects, SM/CG method.**

I. INTRODUCTION

DUE TO THE continuous increase of the operating frequency of printed circuits, the full-wave electromagnetic characterization of microstrip interconnects plays an important role in electronic packaging. A variety of numerical methods have been developed in the past for the electromagnetic simulation of the microstrip interconnects. One effective and popular approach is the spatial-domain mixed-potential integral-equation (MPIE) method in conjunction with the Rao–Wilton–Glisson (RWG) triangular discretization [1] and solving the integral equation by the method of moments (MoM). The employment of the RWG basis function provides a good flexibility to model arbitrarily shaped interconnects. Conventional implementation of the MoM requires $O(N^3)$ operations and $O(N^2)$ computer memory storage, where N denotes the number of unknowns. To provide more functionality and to reduce the size of the package, microstrip interconnects are often packed densely, making the number of unknowns very large. Furthermore, calculation of matrix elements requires the evaluation of the Sommerfeld integral, which involves a highly oscillatory and slowly decaying kernel rendering inefficient computation of the impedance matrix. These make the conventional MoM implementation difficult when the problem scale becomes large. Several fast and efficient methods have been developed to improve the efficiency of the MoM by reducing the number of computation operations and memory requirement, such as the conjugate-gradient fast-Fourier-transform (CG–FFT) method [2], [3], the fast multipole method (FMM) [4], [5], the adaptive integral method (AIM) [6], [7], and the sparse-matrix/canonical-grid (SM/CG) method. The computational complexity and the memory requirement are reduced to $O(N \log N)$ and $O(N)$, respectively, in these fast algorithm.

Over the last few years, we have made significant progress on the SM/CG method for solving large-scale random rough surface scattering problems up to 1.5 million unknowns [8]–[10]. This method entails the decomposition of the impedance matrix into strong near-field interaction and weak far-field interaction matrices. The far interactions, which require most of the computation time in the matrix–vector multiplication (MVM) of an iterative solution, can be computed simultaneously via fast Fourier transforms (FFTs) if the Taylor series expansions of the Green's functions are performed about a uniformly spaced canonical grid. Depending on the number of Taylor series terms, the number of FFTs required in each iteration may still be too large for efficient computation. To alleviate this difficulty, some of the Taylor series terms are moved to the other side of the matrix equation. This is the key difference between this method and the similar AIM developed by Bleszynski *et al.* [6]. We have also successfully implemented the SM/CG method to analyze microstrip structures with RWG triangular discretization by transferring the triangular elements to a uniform grid through expansion of the Green's function to a Taylor series about the uniform grid [9].

In the MPIE formulation, one needs the spatial-domain Green's functions for the magnetic vector potential and the electric scalar potential. For layered media, these spatial-domain Green's functions are often expressed by their corresponding spectral counterparts in the form of Sommerfeld integrals. To evaluate the Sommerfeld integral, several techniques have been developed. One approximation approach is the complex image method (CIM) [11], [12]. In this method, the spectral-domain Green's function is approximated by a

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S.-Q. Li, C. H. Chan, and K. F. Chan are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong.

Y. Yu is with the Center of Computational Electromagnetics, University of Illinois at Urbana-Champaign, Urbana-Champaign, IL 61801 USA.

L. Tsang is with the Department of Electrical Engineering, University of Washington, Seattle, WA 98195-2500 USA.

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sum of complex exponential terms, and these terms are then transformed to the spatial domain using the Sommerfeld identity, which results in a closed-form expression for the spatial-domain Green's function. As stated in [13], the successful use of the CIM requires one to study in advance the spectral-domain behavior of the Green's function in order to decide on the approximation parameters, such as the number of sampling points and the maximum value of the sampling range, when using a one-level CIM approach. The two-level approach [13] is developed to provide more robust and accurate results. Recently, the fast Hankel transform (FHT) method has been successfully employed to calculate the spatial-domain Green's functions in multilayered structures [14]. This is an efficient numerical method to obtain the spatial-domain Green's functions. In the FHT method, the Sommerfeld integration is performed by a discrete convolution. The results of the integration can be considered as the system response of a Hankel filter. The filter function has an explicit series representation in which the series coefficients decrease exponentially, making it possible to evaluate them to any desired accuracy. Also, the error decreases exponentially as the sampling density increases, which means that even a moderate increase in sampling density will make the error decrease drastically. These good features ensure that the FHT approach is robust and efficient.

In our previous paper [9], the CIM method was applied to obtain the spatial-domain Green's function, which includes the following three parts:

1) quasi-dynamic terms;
2) surface-wave terms;
3) complex image terms.

In each part, there are typically 3–4 terms, resulting in a total of over ten terms to represent the Green's function. To combine with the SM/CG method, each term is then expanded by Taylor series expansion leading to the use of the FFTs. In this paper, the FHT method is applied to obtain the spatial-domain Green's function. The Green's function obtained from the FHT method is in numerical form, therefore, it is not suitable for the SM/CG method since it is inconvenient to perform the Taylor series expansion numerically. To give analytical expression to the Green's function, the numerical results of the FHT and the quasi-dynamic and surface-wave contributions are combined together by representing them as a sum of complex exponentials, which can be achieved by using the matrix pencil (MP) method [15]. In general, seven complex exponential terms are sufficient to match the spatial-domain Green's function. The Taylor series expansions of these complex exponentials are then incorporated into the sparse-matrix/canonical grid (SM/CG) method. Associated with the good features of the FHT, a fewer number of terms to evaluate the Green's function on the canonical grid are needed in the algorithm of this paper, which makes the method more efficient. The algorithm is implemented on a parallel computing platform with a cluster of 16 personal computers (PCs). These make the proposed algorithm available to analyze densely packed interconnects efficiently. Numerical examples are presented to demonstrate the validity, accuracy, and efficiency of the proposed method.

II. Formulation

Using the MPIE method to model the microstrip interconnects, the excited field on the microstrip and the vector and scalar potentials are related as [16]

$$\vec{E}^{\rm inc} = j\varpi\vec{A} + \nabla\Phi \tag{1}$$

where the vector and scalar potentials can be expressed by the unknown surface current and the corresponding spatial Green's functions

$$\vec{A}(\vec{r}) = \int_S dS' \overline{\overline{G}}_a(\vec{r}, \vec{r}\,') \cdot \vec{J}(\vec{r}\,') \tag{2}$$

$$\Phi(\vec{r}) = \int_S dS' G_q(\vec{r}, \vec{r}\,') \nabla' \cdot \vec{J}(\vec{r}\,'). \tag{3}$$

The spatial-domain Green's functions in the above equations can be obtained from their spectral counterparts $\tilde{G}_{a,q}$ through the Hankel transform as

$$G_{a,q}(\rho) = \int_{-\infty}^{+\infty} \tilde{G}_{a,q}(k_\rho) H_0^{(2)}(k_\rho \rho) k_\rho \, dk_\rho \tag{4}$$

where $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind, and ρ is the radial separation between the source and observation points in the spatial domain. Applying the RWG basis function to (1), produces the matrix equation

$$[Z]I = V. \tag{5}$$

For densely packed microstrip interconnects, the number of unknowns will be comparatively large. The numerical integration of the Sommerfeld integral of (4) is also time consuming. It is a challenge to solve (5), as we have to address both the issues of matrix fill time and matrix solution time. To solve the problem efficiently, in this paper, the fast SM/CG method is used to solve the matrix equation, in which the computational complexity and memory requirement are $O(N \log N)$ and $O(N)$, respectively, due to the use of FFTs to calculate the MVMs. The efficient numerical FHT algorithm is applied to compute the Sommerfeld integral.

A. SM/CG Method

In the SM/CG method [9], the matrix equation (5) is solved in an iterative manner. The impedance matrix is decomposed into the sum of a sparse matrix $[Z^s]$, denoting the strong neighborhood interactions, and a dense matrix $[Z^w]$, denoting the weak far interactions. Through Taylor expansion of the Green's functions, the matrix $[Z^w]$ is further written as

$$[Z^w] = \sum_{i=0}^{K} [Z_i^w] \tag{6}$$

where K is the total number of terms of the expansion. The iterative procedure is, for the zeroth- and high-order solutions

$$\left\{[Z^s] + [Z_0^w]\right\} I_0 = V \tag{7}$$

$$\left\{[Z^s]+[Z_0^w]\right\}I_{n+1}=V-\left\{\sum_{i=1}^{K}[Z_i^w]\right\}I_n. \quad (8)$$

Due to the translationally invariable kernels in the Green's functions, the weak-matrix vector multiplication can be efficiently performed via the FFTs.

B. Closed-Form Spatial-Domain Green's Functions from the FHT

When using the FHT algorithm to calculate the Sommerfeld integral, the integral is reduced to a discrete convolution and the result is the response of a Hankel filter. Before applying the FHT, the real poles of $\tilde{G}_{a,\,q}$ must be found and extracted since in the FHT method, the integration path is along the real axis. The contributions of these poles can be calculated by residue calculus. After extracting the poles and some quasi-dynamic terms, which can further smooth the Sommerfeld integrand, (4) can be written as

$$G_{a,\,q}^e(\rho)=2\int_0^{+\infty}\tilde{G}_{a,\,q}^e(k_\rho)J_0\big(k_\rho\rho\big)k_\rho\,dk_\rho \quad (9)$$

where J_0 is the zeroth-order Bessel function. The superscript e represents the Green's function after extracting the poles on the real axis and quasi-dynamic terms. The numerical results of the above integral are then obtained by the FHT algorithm, which has been described in detail in [17] and is not given here.

In the FHT algorithm, the spectral-domain Green's function is sampled exponentially, which means that the sample will be very dense for small k_ρ. The Green's function may have sharp peaks and fast changes when k_ρ is small in the spectral domain, which maps to the far-field region in the spatial domain [13]. Compared with the CIM, in which the sampling is uniform, the dense sampling in the FHT algorithm when k_ρ is small can grasp the fast changes and, therefore, can provide more robust and accurate results for the far-field region in the spatial domain.

To obtain analytical expressions of spatial Green's functions from the numerical results of the FHT, we approximate them by a sum of complex exponentials using the well-known MP method [15] since the functions are always limited in a finite spatial range and their arguments are real. The MP method is more computationally efficient and robust at approximating a function by a sum of complex exponentials than other methods, such as the Prony's method and the pencil-of-functions approach. However, in the MP method, the sampling points are required to be uniform, although the direct results of the FHT are exponentially sampled. To obtain a uniform sampled sequence, we apply the same interpolating function used in the FHT algorithm to obtain the spatial-domain Green's functions at arbitrary distance

$$G_{a,\,q}^e(\rho)=\frac{1}{\rho}\sum_{-\infty}^{+\infty}G^*(m\,\Delta)P\left(\frac{\ln\rho}{\Delta}-m\right) \quad (10)$$

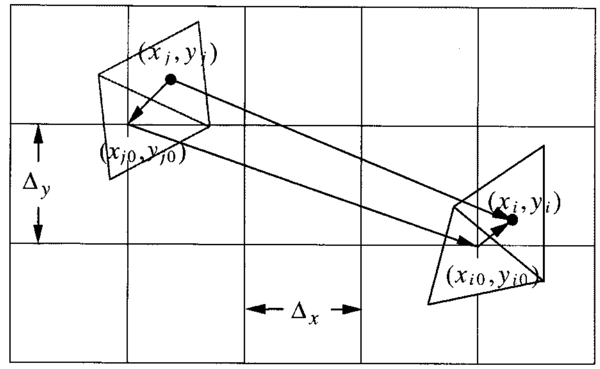


Fig. 1. Direct and indirect computation of the interaction of a pair of triangles on a canonical grid.

in which G^* is the discrete numerical result of the FHT and Δ is the sampling interval. The interpolating function $P(u)$ is defined as

$$P(u)=\frac{a\sin(\pi u)}{\sinh(a\pi u)} \quad (11)$$

where a is a smoothing parameter. Using (10), any desired uniform sampled sequence can be obtained. To reduce the total number of terms in the closed-form expression of the Green's function and, therefore, the number of FFTs in the SM/CG algorithm, the surface-wave and quasi-dynamic contributions are combined together with (10) to obtain the whole spatial analytical Green's function by applying the MP method. The expression as a sum of complex exponentials is

$$G_{a,\,q}(\rho)=\sum_{i=1}^{N}R_i\exp\Big[S_i(\rho-\rho_{\min})\Big] \quad (12)$$

where N is the number of complex exponential terms needed to simulate the numerical sequence well, R_i and S_i are the coefficients of the complex exponential terms, and $\rho_{\min}$ is the minimum distance in the FHT algorithm because the term $\ln\rho$ is used in the FHT algorithm and the result is not available when $\rho=0$. $\rho_{\min}$ can reach a very small value. When it is sufficiently small, we can simply use the quasi-dynamic contributions to approximate the Green's function for $0\le\rho\le\rho_{\min}$.

C. Taylor Series Expansion of the Spatial Green's Function

As stated in [18], if the ratio of the maximum side of the two interacting triangles to the separation of their centroids is below 20%, a point-to-point evaluation of the Green's function weighted by the areas of the triangles is sufficient to approximate the Galerkin procedure. Efficient evaluation of the far-interaction contributions in the MVM is reduced to efficient convolution between the Green's function and current vector. In the SM/CG algorithm applied to microstrip interconnects with triangular discretization, the Taylor expansion of the Green's function is performed as depicted in Fig. 1. The Green's function between two points (x_j, y_j) and (x_i, y_i), which are the centroids of

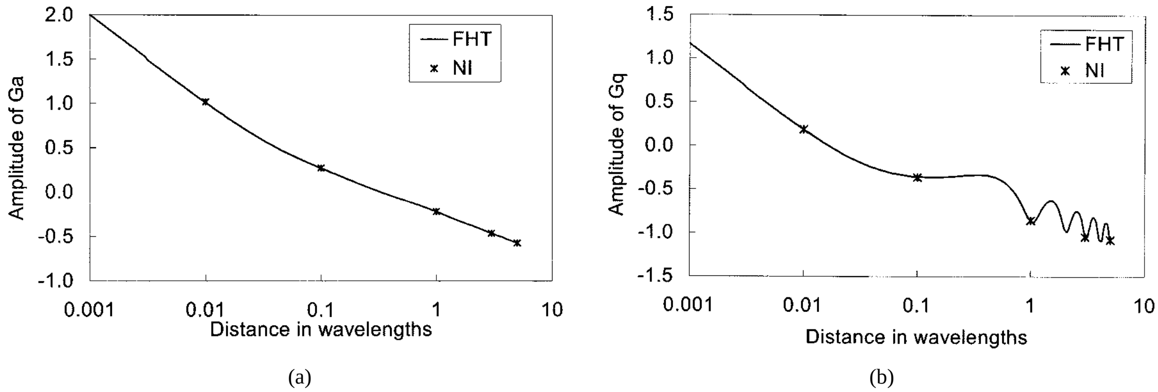


Fig. 2. Comparisons between the results for the spatial Green's functions by the FHT and numerical integration. (a) Amplitude of the vector potential $G_a(\rho)$. (b) Amplitude of the scalar potential $G_q(\rho)$. Permittivity of the dielectric substrate $\varepsilon_r = 12.6$, thickness $h = 1$ mm, frequency $f = 30$ GHz.

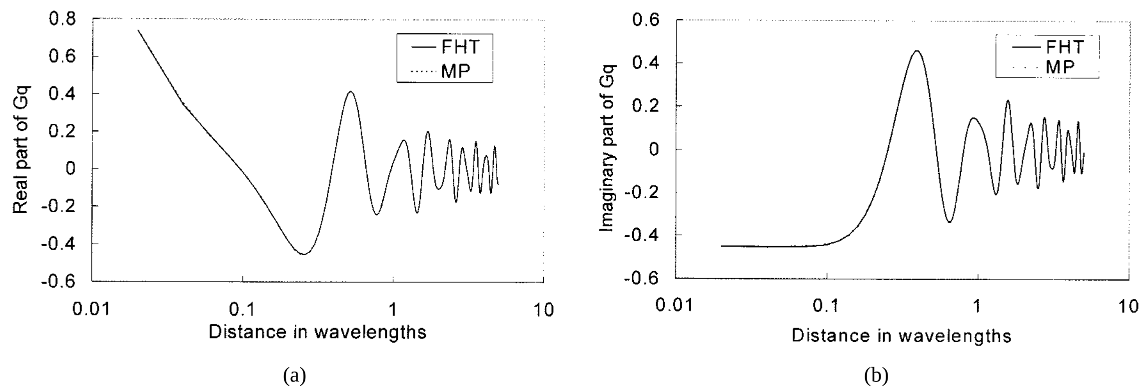


Fig. 3. Comparisons between the numerical results of the FHT and the analytical results of the MP method. (a) Real part of $G_q(\rho)$. (b) Imaginary part of $G_q(\rho)$. Permittivity of the dielectric substrate $\varepsilon_r = 12.6$, thickness $h = 1$ mm, frequency $f = 30$ GHz.

two interacting triangles, can be evaluated indirectly, first from (x_j, y_j) to its nearest grid point (x_{j0}, y_{j0}), then from (x_{j0}, y_{j0}) to another grid point (x_{i0}, y_{i0}), and finally from (x_{i0}, y_{i0}) to (x_i, y_i). This indirect computation corresponds to Taylor series expansion about the canonical grid point at (x_{i0}, y_{i0}). This can be performed symbolically using software such as Maple[1]

$$G(\rho_{i,j}) = \sum_{m_1}\sum_{n_1}\sum_{m_2}\sum_{n_2} \frac{x_{d_i}^{m_1}}{m_1!}\frac{y_{d_i}^{n_1}}{n_1!}\frac{x_{d_j}^{m_2}}{m_2!}\frac{y_{d_j}^{n_2}}{n_2!} \cdot \frac{\partial^{m_1+m_2}}{\partial x_{i0}^{m_1+m_2}}\frac{\partial^{n_1+n_2}}{\partial y_{i0}^{n_1+n_2}} G\left(\sqrt{x_{d_{ij}}^2 + y_{d_{ij}}^2}\right). \quad (13)$$

Substituting (12) into (13), the Taylor's series expansion can be obtained analytically. The evaluation of the MVM for the far-interaction contributions can then be read as [9]

$$[Z^w]I_n = \left\{\sum_{i=0}^{K}[Z_i^w]\right\}I_n = \left\{\sum_{i=0}^{K}[T_{t_i}][G_i][T_{s_i}]\right\}I_n \quad (14)$$

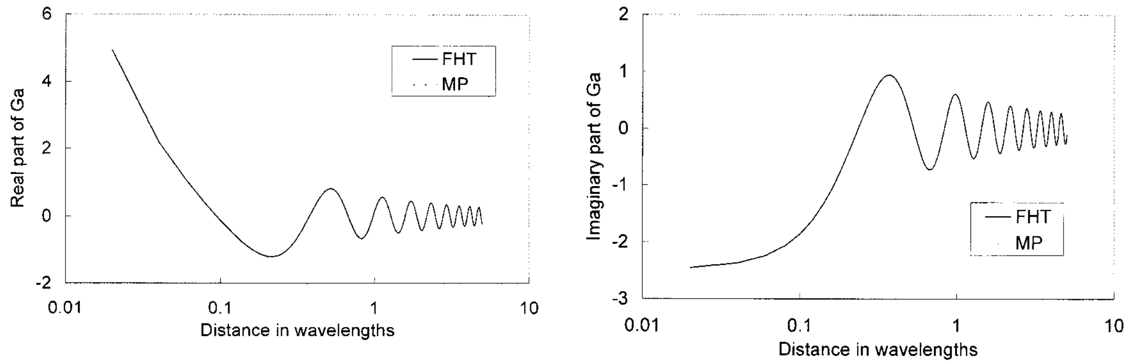
where the block-diagonal matrix $[T_s]$ corresponds to a pre-multiplication, while the other block-diagonal matrix $[T_t]$ corresponds to a post-multiplication. Note that the pre-multiplication corresponds to shifting the centroids of the basis triangles to their nearest grid points. The multiplication of the block-Toeplitz matrix corresponds to computing all the interactions among the uniformly canonical grid points. The final post-multiplication corresponds to translating the interactions at the grid points back to the centroids of the testing triangles. The multiplication with the block-Toeplitz matrix can be then performed by FFTs.

[1]Maple is a registered trademark of Waterloo Maple Software, Waterloo, ON, Canada.

III. Numerical Results

Before applying the FHT algorithm, to prove its validity, the comparisons between the results obtained by the FHT algorithm and those obtained by a direct numerical integration are given in Fig. 2(a) and (b) for the spatial Green's functions of the vector and scalar potentials G_a and G_q, respectively. The direct numerical integration results are denoted by the notation of "NI" in the figures. The microstrip structure considered is a single-layered dielectric substrate with ground plane. The thickness and dielectric constant of the substrate are $h = 1$ mm and $\varepsilon_r = 12.6$, and the operation frequency is $f = 30$ GHz. The direct numerical integration is time consuming and the Green's functions are calculated in only a few points. However, for the FHT algorithm, a large data sequence up to 300 different locations can be obtained in a few seconds on a Pentium MMX 233 PC. It can be seen from the figure that the FHT results agree well with those obtained by direct numerical integration.

Figs. 3 and 4 give the comparisons between the numerical results of the FHT and the analytical results of the MP method for the vector potential $G_a(\rho)$ and the scalar potential $G_q(\rho)$,



(a) (b)

Fig. 4. Comparisons between the numerical results of the FHT and the analytical results of the MP method. (a) Real part of $G_a(\rho)$. (b) Imaginary part of $G_a(\rho)$. Permittivity of the dielectric substrate $\varepsilon_r = 12.6$, thickness $h = 1$ mm, frequency $f = 30$ GHz.

TABLE I

COEFFICIENTS OBTAINED BY THE MATRIX PENCIL METHOD FOR FIGS. 3 AND 4 WHERE $\rho_{\min} = 0.02\ \lambda_0$, AND λ_0 IS THE FREE-SPACE WAVELENGTH

G_a		G_q	
R_i	S_i	R_i	S_i
1.98649678E-01 -j2.68768944E-01	-5.92961664E-02 -j1.04468009E+01	-2.33506784E-02 +j5.19854948E-02	-1.18638170E-01 -j1.65569056E+01
2.14481632E-01 -j3.30789091E-01	-6.02740275E-01 -j1.04849367E+01	4.79107187E-02 -j2.33306361E-01	-1.37366352E-01 -j1.03542319E+01
2.97952599E-03 +j1.09885799E-02	-9.20779501E-01 -j5.91494480E+00	6.62071631E-02 +j3.15331622E-02	-2.26041477E-01 -j1.00544855E+01
2.71402774E-01 -j5.33903279E-01	-2.24105778E+00 -j1.03397092E+01	-2.43300046E-02 -j7.97790978E-03	-9.32259980E-01 -j5.69177898E+00
1.06517881E+00 -j7.24017738E-01	-9.04551702E+00 -j9.14739428E+00	-2.37257004E-02 +j7.51757375E-02	-1.30378583E+00 -j1.63846967E+01
1.56054385E+00 -j2.35037804E-01	-3.24670142E+01 -j1.12322684E+01	2.39326696E-01 -j2.65438672E-01	-2.42149778E+00 -j1.04341319E+01
1.61224918E+00 -j3.69420108E-01	-9.51377390E+01 -j1.08120762E+01	4.53930203E-01 -j1.02221850E-01	-5.49750530E+01 -j1.22873125E+01

respectively. Seven exponential terms are used to fit the numerical results of the FHT. It is found that the two results agree very well. The coefficients obtained by the MP method are tabulated in Table I.

To validate our result, we calculate the scattering parameters of a microstrip stub, for which measurement results are available in the literature, and compare the measurement results with the computed results obtained by the SM/CG method associated with the FHT algorithm. The geometry and discretization of the simulated microstrip stub are given in Fig. 5. The operation frequency is $f = 7.5$ GHz. For the analysis using the triangular MoM, the two ports extend 40 cells on each side in order to obtain a sufficient number of current samples. The bilateral symmetry in the stub is exploited in processing the port current data. The computed current on the main line of this microstrip stub obtained by the SM/CG with the FHT and that by a direct matrix solution method in [19] are given in Fig. 6(a). The measured and the computed results of S_{12} are shown in Fig. 6(b). The measurement data are taken from [20]. Good agreement between the computed and measured results confirms the validity of our method.

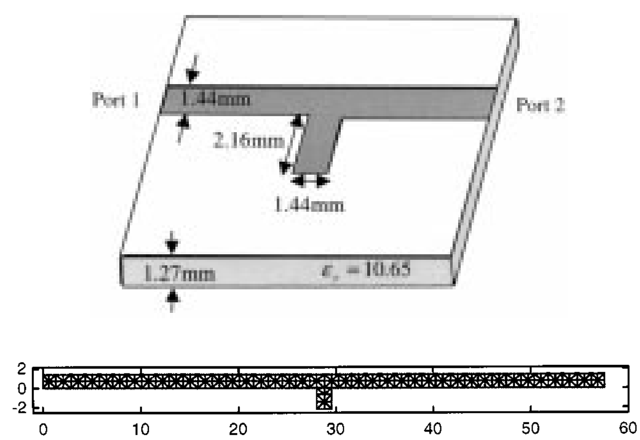


Fig. 5. Geometry and discretization of a microstrip stub.

A group of 12 densely packed curved microstrip lines is studied. The top view of these lines and their triangular discretization is shown in Fig. 7(a). The thickness and dielectric constant of the substrate are $h = 1$ mm and $\varepsilon_r = 12.6$. The operation frequency is $f = 30$ GHz. The horizontal dimension of each line is 6.2 mm and the width is 0.3 mm. The lines are separated by 0.2 mm. The canonical grid is set at $dx = dy = 0.1$ mm, which is about 30 points per linear dielectric wavelength. Here a ten-term Taylor series expansion is used. We excite only the first and the twelfth lines in Fig. 7(a)

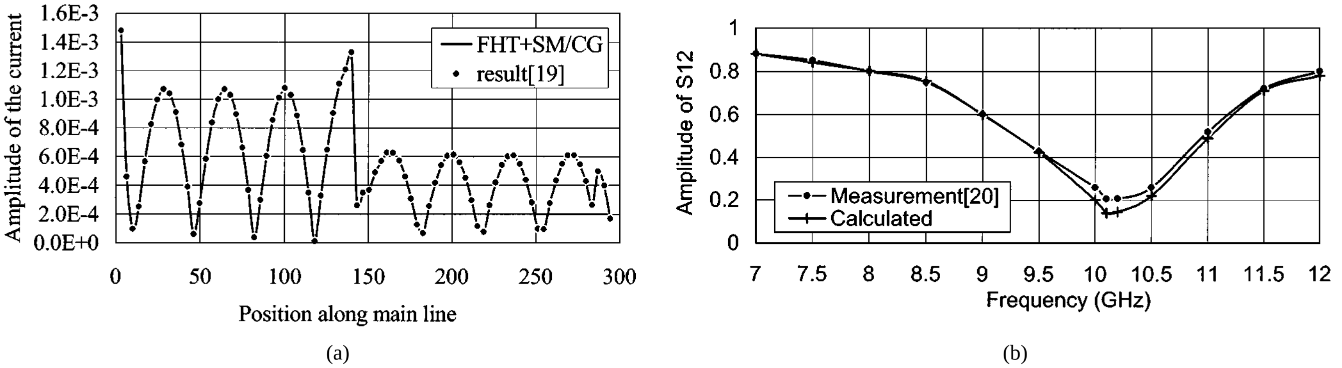


Fig. 6. Comparison between the results of a microstrip stub by the SM/CG and direct solution [19]. (a) Current distribution along the main line of the microstrip stub (frequency is 7.5 GHz). (b) Transmission response of the microstrip stub: measured [20] and computed by the SM/CG with the FHT.

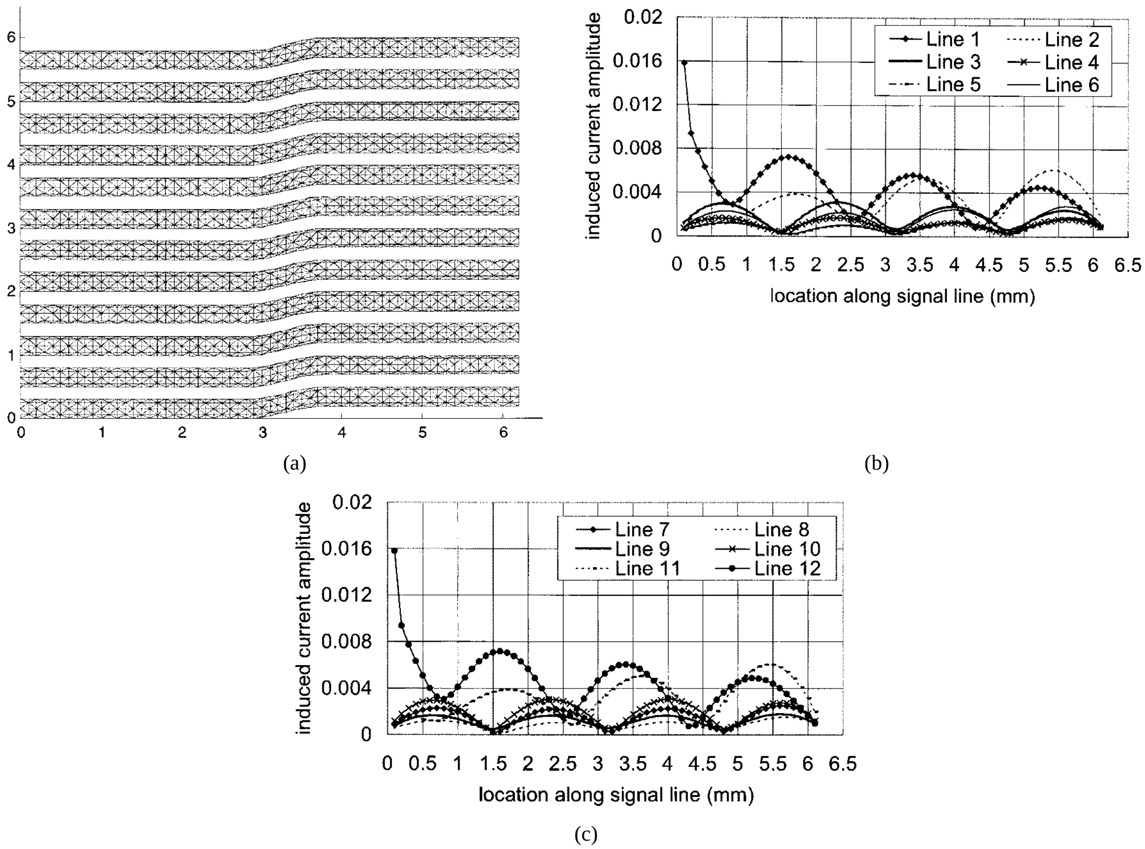


Fig. 7. Densely packed curved signal lines. (a) Geometry. (b) Current distributions on Line 1–Line 6. (c) Current distributions on Line 7–Line 12.

and observe the mutual coupling effects on the rest of the lines. There are 8136 unknowns in this example. These parallel signal lines are numbered from the bottom to top. The number of the exponential summation in the MP method is selected as seven. The induced current distributions on these lines are shown in Fig. 7(b) and (c). From these figures, it is noted that the excitation on the first and twelfth lines can induce different current distributions on other signal lines. The total CPU time for this problem is only about 5 min on a cluster of 16 PCs [21]. The Message Passing Interface (MPI) [22] and the MPI version of the Fast Fourier Transform in the West (FFTW)[2] [23] are employed in the parallel computer code. Each PC has a Pentium II 450-MHz processor with 256-MB RAM. Fig. 8 shows the convergence of the Bi-CGM for each order of the proposed SM/CG method. The zeroth-order solution corresponds to solving (7) while the subsequent orders correspond to that of (8). Note that the number of iterations required is reduced substantially when the order of the solution increases. Fig. 9 shows the convergence of the percentage error of the SM/CG method. It can be seen that the percentage error is also reduced substantially when the order of the iterative solution increases.

In our last numerical example, a very large-scale interconnect in which the number of unknowns is over 36 000 is studied. The top view of the geometry is shown in Fig. 10. The thickness and dielectric constant of the substrate and operation frequency are the same as those in the above example. The horizontal and vertical dimensions are 6.2 and 6.1 mm, respectively. Since the width of the small sections on both sides of the interconnect and

[2]Free FFTW 2.1.2 manual download. [Online]. Available: http://www.fftw.org

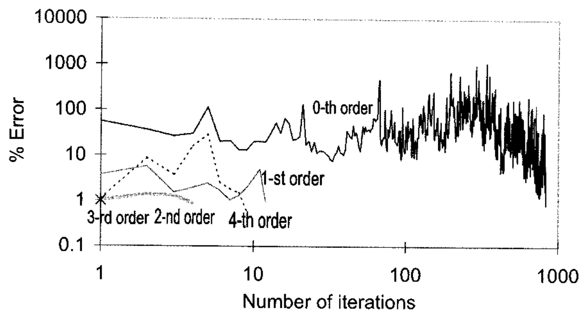


Fig. 8. Convergence of the Bi-CGM for each order of the SM/CG method for the example shown in Fig. 7.

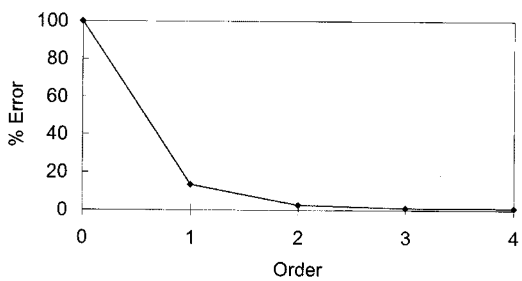


Fig. 9. Convergence of the SM/CG method for the example shown in Fig. 7.

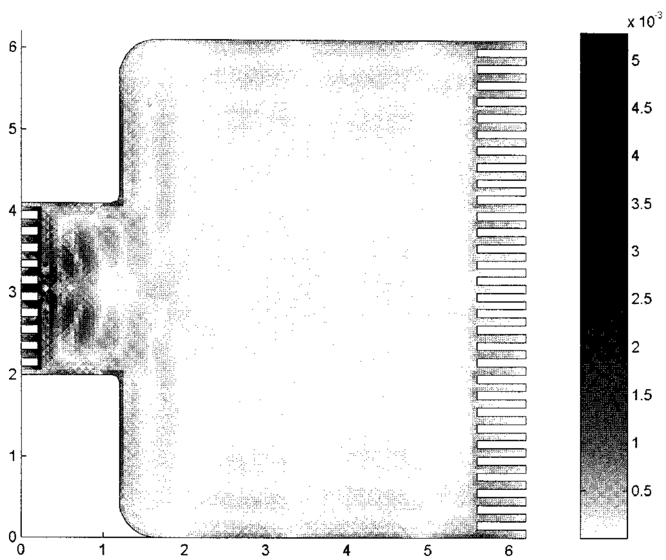


Fig. 10. Top view of the large-scale interconnect and the current distribution on it.

the separation between adjacent small sections are only 0.1 mm, the canonical grid is set at $dx = dy = 0.05$ mm, resulting in a total of 36 578 unknowns. The interconnect is excited in the middle of the left-hand side. The total CPU time for this problem is only about 30 min when a cluster of eight PCs is used. The result of the current distribution on the interconnect is also shown in Fig. 10.

IV. Conclusion

In summary, we have presented an SM/CG method for the analysis of densely packed interconnects. The method entails the use of Taylor series expansions of the spatial Green's functions obtained by the FHT and MP method. The majority of the interactions among the current elements are computed simultaneously using FFTs. It requires much less CPU time and memory when compared with the conventional conjugate gradient iterative solver. Due to the use of the FFT, the proposed method is particularly suitable for parallel computing platforms.

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Shu-Qing Li (M'00) received the B.S. and M.S. degrees in radio science from Shandong University, Jinan, China, in 1991, and the Ph.D. degree in electronic engineering from Xi'an Jiaotong University, Xi'an, China, in 1994 and 1998, respectively.

In September 1998, she joined the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, as a Senior Research Assistant, and became a Research Fellow in September 2000. Her current research interests include computational electromagnetics, microwave remote sensing, high-frequency interconnects, and parallel computing.

Yongxue Yu received the B.S. and M.S. degrees in electronic engineering from Sichuan University, Chengdu, China, in 1990 and 1993, respectively, and the Ph.D. degree from the City University of Hong Kong, Kowloon, Hong Kong, in 1999.

From August 1998 to August 1999, she was a Research Assistant in the Wireless Communications Research Center, City University of Hong Kong. She is currently a Post-Doctoral Research Associate in the Center of Computational Electromagnetics, University of Illinois at Urbana-Champaign. Her research interests include electromagnetic characterization of interconnects for electronic packaging, and analysis and simulation of interconnects and periodic structures.

Dr. Yu was the recipient of Hong Kong Association of University Women Postgraduate Scholarship (1997–1998).

Chi Hou Chan (S'86–M'86–SM'00) received the Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign, in 1987.

From 1987 to 1989, he was a Visiting Assistant Professor in the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. In 1989, he joined the Electrical Engineering Department, University of Washington, Seattle, as an Assistant Professor, and then became an Associate Professor in 1993. In April 1996, he joined the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong, as a Professor, and became a Professor (Chair) of electronic engineering in 1998. Since 1998, he has been the Associate Dean (Research) of Faculty of Science and Engineering at the City University of Hong Kong. He is a Guest Professor at Xi'an Jiaotong University, Xi'an, China. He authored "MULTFSS," a general-purpose frequency-selective surface analysis code currently distributed by DEMACO Inc., Champaign, IL, as part of its McFSS package. His research is focused on computational electromagnetics and wireless communications.

Prof. Chan is a Fellow of the Chinese Institute of Electronics (CIE) and the Institute of Electrical Engineers (IEE), U.K. He was a recipient of the 1991 U.S. National Science Foundation Presidential Young Investigator Award.

Ka Fai Chan was born in Hong Kong, on October 12, 1975. He received the B.Eng. degree in electronic engineering from the City University of Hong Kong, Kowloon, Hong Kong, in 1999, and is currently working toward the M.Phil. degree at the City University of Hong Kong.

Leung Tsang (S'73–M'75–SM'85–F'90) was born in Hong Kong. He received the B.S., M.S., and Ph.D. degrees from the Massachusetts Institute of Technology, Cambridge, in 1971, 1973, and 1976, respectively.

He is currently a Professor of electrical engineering at the University of Washington, Seattle. He co-authored *Theory of Microwave Remote Sensing* (New York: Wiley, 1985). His current research interests include wave propagation in random media and rough surfaces, remote sensing, optoelectronics, and computational electromagnetics.

Dr. Tsang is a Fellow of the Optical Society of America. He has been the Editor-in-Chief of the IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING since 1996. He was the Technical Program chairman of the 1994 IEEE Antennas and Propagation Society (IEEE AP-S) International Symposium and URSI Radio Science Meeting, the Technical Program chairman of the 1995 Progress in Electromagnetics Research Symposium, and the general chairman of the 1998 IEEE International Geoscience and Remote Sensing Symposium. He was the recipient of the 2000 Outstanding Service Award presented by the IEEE Geoscience and Remote Sensing Society. He was also the recipient of the IEEE Third Millennium Medal.